

Non Perturbative One Gluon Exchange Potential from Dyson-Schwinger Equations

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Recent progress in the solution of Dyson-Schwinger equations of QCD allows for a non perturbative evaluation of the One Gluon Exchange (OGE) interaction. We calculate the interquark static potential for heavy mesons by assuming that it is given by this OGE interaction and we apply it to the description of charmonium.

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1 Introduction

The development of non-perturbative techniques in Quantum Chromodynamics (QCD) is starting to allow for a description of the hadron spectrum from first principles. Among these techniques Lattice gauge theory [1, 2] constitutes a non-perturbative regularisation scheme which may provide accurate numerical solutions of the theory. The accuracy of lattice results has been tremendously improved during the past decade with the availability of more powerful computers [3, 4]. Therefore, lattice results are considered in many instances the data which other non perturbative approximations try to reproduce. One of these alternatives is the approximate resolution of the Dyson-Schwinger Equations (DSE) of QCD, a non-perturbative approach which has progressed considerably in the last ten years. This approach, more analytical, has led to a very appealing physical picture establishing that the QCD running coupling (effective color charge) freezes in the deep infrared. This property can be best understood from the point of view of a dynamical gluon mass generation that is a purely non-perturbative effect [5, 6, 7].

The aim of this presentation is to investigate, following reference [8], the form of the OGE static potential from DSE and compare it to the static potentials derived from lattice calculations. The application of these potentials to the description of quarkonia will be discussed.

2 One Gluon Exchange Potential from Dyson Schwinger Equations

It is well established by now that the QCD running coupling (effective charge) freezes in the deep infrared what may be understood in terms of a dynamical gluon mass. Even though the gluon is massless at the level of the fundamental QCD Lagrangian, and remains massless to all order in perturbation theory, the non-perturbative QCD dynamics generate an effective, momentum-dependent mass, without affecting the local $SU(3)_c$ invariance, which remains intact [5, 6, 7, 9]. At the level of the Dyson-Schwinger equations (DSE), solved by using a PT (pinch technique) - BFM (background field method) truncation scheme in the quenched approximation (no quark loops), the generation of such a mass is associated with the existence of infrared finite solutions for the gluon propagator. Such solutions may be fitted by “massive” euclidean propagators of the form $\Delta^{-1}(q^2) = q^2 + m^2(q^2)$ where $m^2(q^2)$ depends non-trivially on the momentum transfer q^2 .

One physically motivated possibility is the so called logarithmic mass running, which is defined by

$$m^2(q^2) = m_0^2 \left[\ln \left(\frac{q^2 + \rho m_0^2}{\Lambda^2} \right) / \ln \left(\frac{\rho m_0^2}{\Lambda^2} \right) \right]^{-1-\delta}. \quad (1)$$

where Λ is the QCD scale and m_0 , the gluon mass at $q^2 \rightarrow 0$, as well as ρ and δ are constants to be fitted. For this fitting the calculated DSE propagator is compared to the one obtained from lattice.

On the other hand the non-perturbative generalization of $\alpha(q^2)$, the QCD running coupling, comes in the form

$$\alpha(q^2) = 4\pi \left[\beta_0 \ln \left(\frac{q^2 + \rho m^2(q^2)}{\Lambda^2} \right) \right]^{-1}, \quad (2)$$

where $\beta_0 = 11 - 2n_f/3$ being n_f the number of active quark flavours.

Note that its zero gluon mass limit leads to the LO perturbative coupling constant momentum dependence. The $m(q^2)$ in the argument of the logarithm tames the Landau pole, and $\alpha(q^2)$ freezes at a finite value in the IR, namely $\alpha(0) = 4\pi \left[\beta_0 \ln \left(\frac{\rho m_0^2}{\Lambda^2} \right) \right]^{-1}$.

From these expressions for the propagator and the coupling we can derive the One Gluon Exchange interaction between static charges, see Fig 1.

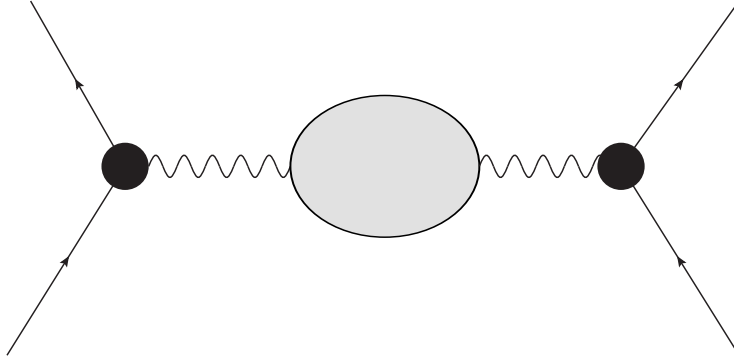


Figure 1: One gluon exchange interaction.

The OGE static potential is related to the Fourier transform of the time-time component of the full gluon propagator in the following way

$$V(\mathbf{r}) = -C_F \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{4\pi\alpha(\mathbf{k}^2)}{\mathbf{k}^2 + m^2(\mathbf{k}^2)} e^{i\mathbf{k}\cdot\mathbf{r}} = -\frac{32\pi C_F}{\mathbf{r}} \int_0^\infty d\mathbf{k} \mathbf{k} \frac{\alpha(\mathbf{k}^2)}{\mathbf{k}^2 + m^2(\mathbf{k}^2)} \sin(\mathbf{k}r) \quad (3)$$

where C_F is the Casimir eigenvalue of the fundamental representation of $SU(3)$ [$C_F = (N^2 - 1)/2N$ for $SU(N)$] and the bold terms, \mathbf{k} and \mathbf{r} stand for 3-vectors.

In Fig. 2 we show the finite part (up to a constant) of the potential, Eq 3, derived from the DSE with the definitions in Eqs 2 and 1. We have chosen the following range of parameters: $m_0 \sim 360 - 480$ MeV, $\rho = 1. - 4.$, $\delta = 1./11$ [5, 10, 11] which provide a good fit to the lattice propagator. The value of Λ here has been taken to be 300 MeV. In order to adjust the behavior at the origin to the data we have used β_0 corresponding to $n_f = 4$ flavors. To do this appropriately one should introduce the running of the quark masses, which are at present not well known. However, since asymptotically the masses run to zero, our way of proceeding achieves the correct (perturbative) strength of the potential at low r . The potential describes well the low radial behavior, by construction, flattens at large r going asymptotically to zero and never becomes positive.

For comparison we have used i) a Cornell type potential (whose form is derived from quenched lattice calculations [12, 13])

$$V(r) = -a/r + br. \quad (4)$$

containing the perturbative expectation plus an additional linear term and ii) a screened type potential (whose form was derived long time ago from an unquenched lattice calculation [14])

$$V(r) = (-a/r + br) \left(\frac{1 - e^{-\gamma r}}{\gamma r} \right) \quad (5)$$

The values used for the parameters are based on spectroscopy (see next Section).

We should realize that the additive infinite self-energy contribution associated with the static sources should be removed from the calculated OGE potential [12]. In lattice QCD this is done normalizing the potential such that $V(r_0) = 0$ where r_0 is the Sommer scale [15] with a typical value around 0.5 fm. We proceed the same way but for phenomenological purposes we take the subtraction point at the zero point of the potentials used in Table 1, which happens to be at $r_0 \sim 0.35 fm$ for the parametrizations we are using. The result of this procedure is shown in Fig. 3. This procedure increases the value of the potential without changing its shape.

As can be checked our OGE potential resembles the screened and not the Cornell potential. We have shown that none of the parameters plays a fundamental role in the structure of the DSE OGE potential, since the structure does not vary when we vary them. All values discussed lead to the same qualitative features for the potential. It is evident that there is no way to reproduce the large r behavior of the Cornell potential by changing the parameters. The DSE OGE potential flattens and becomes basically constant similar to the screened potential. If we assume that the OGE interaction is the main source of the dynamics, this result is quite surprising since the approximations used to find the solution to the DSE do not contain quark loops and therefore they incorporate no mechanism for screening, i.e. some mechanism derived from the breaking of the string [14, 16].

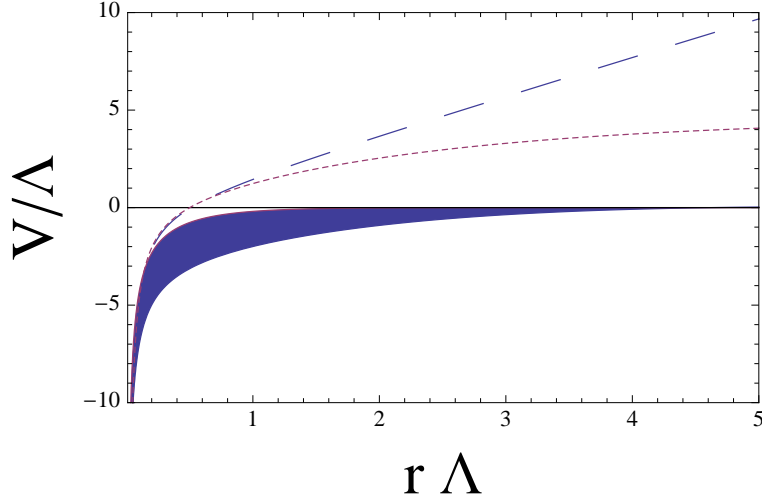


Figure 2: The DSE OGE potential, i.e. the Fourier transform of the massive One Gluon Exchange, is plotted for the range $m \sim 360 - 480$ MeV and $\rho \sim 1 - 4$ with $n_f = 4$ ($\beta_0 = 25/3$). For comparison we plot the Cornell and screened potentials with parameters $a = 0.52$, $\sqrt{b} = 427$ MeV and $\gamma = 0.38$ fm $^{-1}$.

Therefore we should conclude that either some correction to the calculated OGE is lacking (this could come from the use of a higher order truncation scheme for the resolution of the DSE, from vertex corrections to the coupling ...) or that the incorporation of multigluon effects is essential for deriving a consistent large r behavior. In this respect an *ad hoc* explanation based on OGE dominance plus additional coupling corrections has been recently proposed along the following arguments [17]. Let us consider that there were a nonperturbative vertex correction to the strong effective charge in the form

$$\alpha_{conf}(q^2) = \frac{c\Lambda^4}{q^4}$$

with c a constant to be fitted, so that

$$\alpha_{total}(q^2) = \alpha_{conf}(q^2) + \alpha(q^2)$$

where the last term on the right hand side would be given by Eq 2. Then it would be possible to fit not only the lattice propagator but also the Cornell type lattice potential and a fully consistent scheme in the quenched approximation would arise. To go further one should incorporate quark loop effects. According to reference [18] the former vertex correction could be effectively modified. If this modification were parametrized through a cutoff (s) in the form

$$(\alpha_{conf}(q^2))_{unquenched} = \frac{d\Lambda^4}{(q^2 + s^2)^2}$$

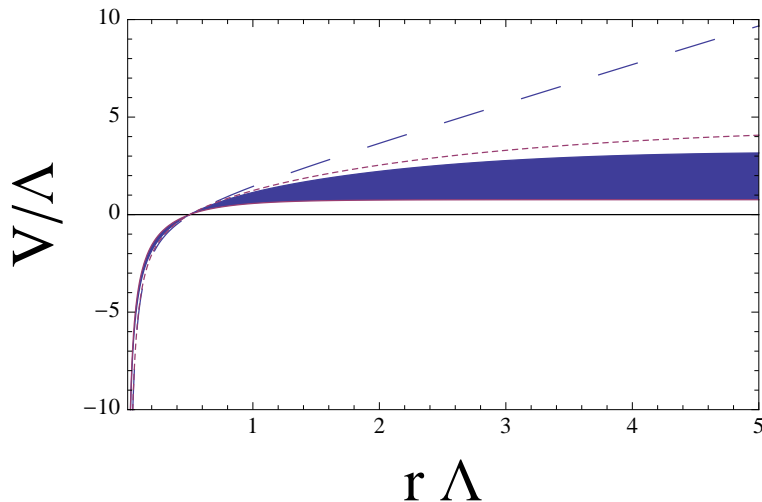


Figure 3: The DSE OGE potential after the Sommer subtraction, with the same parameters as in the previous figure, is plotted. For comparison we plot the Cornell and screened potentials.

being d a constant, the resulting DSE OGE potential would be like the screened potential. In this manner an explanation of our previous results compatible with one gluon exchange dynamical dominance could come out. Next we shall adopt this interpretation in the phenomenological application of the SDSE OGE potential to charmonium.

3 Quarkonia description

For sufficiently heavy quarks, one may hope that the bound state problem becomes essentially non-relativistic the dynamics being controlled approximately by a Schrödinger equation with a static potential. It should be emphasized that the derived potentials do not contain spin-dependent terms what make them more reliable when these terms do not play a major role. This is expected to occur for high excited (large-sized) states since spin corrections are short-ranged. For the low lying states data show that spin-spin splittings between spin triplet and spin singlet states should be relevant ($m_{J/\psi} - m_{\eta_c} = 117$ MeV, $m_{\psi_{2s}} - m_{\eta_{c(2s)}} = 49$ MeV, $m_{\Upsilon(1s)} - m_{\eta_b} = 69$ MeV, ...). By considering that the perturbative spin-spin correction is in absolute value three times bigger for singlets than for triplets, the radial potential approach could be taken as an approximate description of spin triplet states. Other spin-dependent corrections (spin-orbit, tensor) may be playing some role. It is worth to point out that perturbative spin-orbit and tensor splittings cancel in the centroids of p waves

which consequently may be used as “data” for comparison with the radial approach results. Further relativistic corrections are expected to be more important for charmonium than for bottomonium. Therefore the radial approaches are better suited for the study of bottomonium. One should not forget though that in the application to quarkonia the parameters entering the expressions of the potentials have an effective character since their values may be implicitly incorporating non considered corrections.

The charmonium spectrum with well established quantum numbers, corresponding mostly to $J^{PC} = 1^{--}$ resonances produced through ISR (Initial State Radiation) processes, is analyzed in Table 1. The calculated spectrum from the DSE OGE potential is compared to experimental data and to a Cornell like potential calculation (the choice of charmonium instead of bottomonium makes clearer the differences between both calculations). The parameters for the Cornell potential have been chosen within the conventional spectroscopic range $a \sim 0.51 - 0.52$ and $\sqrt{b} \sim 412 - 427$ MeV [20, 21] (note that concerning the description of quarkonia masses the additive constant in the potential can be absorbed in a renormalization of the quark mass).

$n_r L$	$M_{Cornell}$	M_{DSE}	M_{PDG}
	MeV	MeV	MeV
1s	3069	3151	3096.916 ± 0.011
2s	3688	3660	3686.09 ± 0.04
1d	3806	3761	3772.92 ± 0.35
3s	4147	4004	4039 ± 1
2d	4228	4070	4153 ± 3
4s	4539	4273	4263^{+8}_{-9}
3d	4601	4321	$4361 \pm 9 \pm 9$
5s	4829	4487	4421 ± 4
4d	4879	4526	
6s	5218	4651	$4664 \pm 11 \pm 5$
1p	3502	3515	3525.3 ± 0.2
2p	3983	3886	

Table 1: Calculated masses, $M_{Cornell}$ and M_{DSE} , from the Cornell and DSE OGE potentials. For the Cornell potential $a = 0.52$, $\sqrt{b} = 412$ MeV and $m_c = 1350$ MeV. For the DSE potential $m_0 = 345.7$ MeV, $\rho = 1$ and $m_c = 1400$ MeV. Masses for experimental candidates, M_{PDG} , have been taken from [19]. For p waves we quote the centroid of np_0 , np_1 and np_2 states.

As can be checked the main difference between the two models refers to the description of the higher excited states. The DSE model allows in the overall for a reasonable one to one assignment of calculated states to data (within 60 MeV differ-

ence) whereas the Cornell model, providing a good fit for the lower states (at most 30 MeV difference with data), can not accomodate all the known higher energy resonances but only some of them. For instance $\psi(4040)$, $\psi(4160)$ and $\psi(4415)$ may be assigned to the Cornell $3s$, $2d$ and $4s$ states respectively. Then other two resonances, cataloged in the Particle Data Group Review [19] as $X(4260)$ and $X(4360)$, can not be reproduced (this has motivated alternative, non $c\bar{c}$, explanations for these states even though their properties might be understood as corresponding to $c\bar{c}$ states, see [22] and references therein).

Although more complete analyses are needed before extracting any definite conclusion these results seem to point out that the nonperturbative OGE potential might provide a well founded approach to heavy meson spectroscopy.

4 Summary

We have calculated the OGE static potential from an approximate solution of the quenched Dyson-Schwinger equations for the gluon propagator. The low r behavior is determined by the well know asymptotic behavior. The large r behavior is certainly non perturbative. The Sommer procedure, to avoid self energy effects of the static charges, leads to a potential which is not negative everywhere. The DSE with the Sommer normalization is quite similar to a screened potential form derived from unquenched lattice calculations.

This suggests that non considered nonperturbative vertex corrections to the strong effective coupling could be responsible for the linear confining. These corrections might be canceled in the unquenched DSE solution providing an ad hoc explanation of the OGE results obtained.

Taking for granted this explanation and assuming that the OGE interaction is the main source of the dynamics we have proceeded to a calculation of the charmonium spectrum. We have found a one to one correspondence between the calculated states and the experimental resonances. This makes us tentatively conclude that the nonperturbative OGE interaction may provide a significant improvement in the description of heavy quarkonia as compared to conventional potentials based on confinement plus perturbative OGE terms.

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References

- [1] K. G. Wilson, Phys. Rev. D **10**, 2445 (1974).
- [2] M. Creutz, Phys. Rev. D **21**, 2308 (1980). ; arXiv:1103.3304 [hep-lat].
- [3] C. Sachrajda, PoS **LATTICE2010** (2010) 018 [arXiv:1103.5959 [hep-lat]].
- [4] P. Hagler, Prog. Theor. Phys. Suppl. **187**, 221 (2011).
- [5] J. M. Cornwall, Phys. Rev. D **26**, 1453 (1982).
- [6] A. C. Aguilar and J. Papavassiliou, JHEP **0612**, 012 (2006)
- [7] D. Binosi and J. Papavassiliou, Phys. Rept. **479**, 1 (2009) [arXiv:0909.2536 [hep-ph]].
- [8] P. González, V. Mathieu and V. Vento, Phys. Rev. D **84**, 114008 (2011).
- [9] A. C. Aguilar, D. Binosi and J. Papavassiliou, Phys. Rev. D **78**, 025010 (2008).
- [10] A. C. Aguilar and J. Papavassiliou, Eur. Phys. J. A **35**, 189 (2008).
- [11] A. C. Aguilar, D. Binosi, J. Papavassiliou and J. Rodriguez-Quintero, Phys. Rev. D **80**, 085018 (2009);
- [12] G. S. Bali, Phys. Rept. **343**, 1-136 (2001). [hep-ph/0001312].
- [13] J. Greensite, S. Olejnik, Phys. Rev. **D67** 094503 (2003). [hep-lat/0302018].
- [14] K. D. Born, E. Laermann, N. Pirch, T. F. Walsh, P. M. Zerwas, Phys. Rev. **D40** 1653-1663 (1989).
- [15] R. Sommer, Nucl. Phys. B **411**, 839 (1994) [arXiv:hep-lat/9310022].
- [16] S. Kratochvila, P. de Forcrand, Nucl. Phys. **B671**, 103-132 (2003). [hep-lat/0306011].
- [17] V. Vento, arXiv: 1205.2002 [hep-ph].
- [18] J. Papavassiliou and J. M. Cornwall, Phys. Rev. **D44** 1285 (1991).
- [19] K. Nakamura *et al.* [Particle Data Group], J. Phys. G **37**, 075201 (2010).
- [20] C. Quigg, J. L. Rosner, Phys. Rept. **56**, 167-235 (1979).
- [21] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, T. -M. Yan, Phys. Rev. **D21**, 203 (1980).

- [22] E. Eichten, S. Godfrey, H. Mahlke and J. Rosner, Rev. Mod. Phys. 80, 1161 (2008).